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Arbitrary-width confined states of traveling-wave convection: Pinning, locking, drift, and stability

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I describe observations of "arbitrary-width" confined states of traveling-wave convection in an ethanol-water mixture with separation ratio $\psi = -0.253$, made in an extremely uniform annular cell. The time derivative of the spatial width of these confined states vanishes at a unique, width-dependent Rayleigh number, not over a finite band of Rayleigh numbers as previously reported. Also, the confined states drift slowly around the cell, *backwards*.

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Rayleigh-Bénard convection in a thin, horizontal layer of a binary-fluid mixture which is heated from below takes the form of traveling waves (TW's), which are triggered by a subcritical bifurcation from the quiescent state when the Rayleigh number r is increased above a threshold r_{co} [1,2]. Once nonlinear TW's fill the experimental cell, they will persist below onset, giving way to the quiescent state again only when r is reduced below a saddle-node Rayleigh number $r_s < r_{co}$. In the hysteresis loop between r_s and r_{co}, both TW's and the quiescent state are stable solutions of the equations of motion for this system. In the context of the complex Ginzburg-Landau equation (CGLE), a frequently used model of this system, it is possible for these two states to stably coexist, separated in space by fronts, if the system is sufficiently large [3]. This model also has "pulse" solutions; i.e., narrow convective structures surrounded by inactive fluid [3]. In this paper, I refer generally to such localized convecting regions surrounded by quiescent fluid as "confined states."

The first confined state of TW's observed in this system was created in a narrow, rectangular cell [4], and was subsequently shown to closely resemble a pulse solution of a subcritical CGLE [3,5]. Experimentally, TW pulses have been studied at separation ratios ψ in the range $-0.13 \lesssim \psi \lesssim -0.03$. Their spatial structure—in particular, their width—is remarkably insensitive to experimental parameters, and they drift slowly through a sufficiently uniform cell [5–7]. The drift velocity v_{dr} is extremely sensitive to the local Rayleigh number. For most experimental parameters, pulses drift forwards; i.e., in the same direction as the phase velocity of the underlying TW's [6]. Many important features of fixed-width TW pulses have been reproduced in numerical integrations of the full Navier-Stokes equations in two dimensions [8]. While the slow drift cannot be properly modeled using a single-field CGLE, Riecke has recently shown how to self-consistently derive a CGLE coupled to a slow mean concentration field; this coupling causes pulses to drift at slow velocities that are qualitatively consistent with experimental observations [9].

For more negative separation ratios, confined-state behavior in this system is quite different. In experiments at $\psi = -0.25$ reported in Ref. [10], confined states of any width could be created. Once such an "arbitrary-width" confined state was made, its spatial extent could be made time independent by changing the Rayleigh number r to a value inside a narrow band (width 2.3% in [10]) deep inside the hysteresis loop. The confined state could be made to grow (shrink) only by adjusting r above (below) this Rayleigh-number band. Remarkably, inside this "locking band," the width of the confined state remained fixed, and there was no spatial drift, even if r was moved around inside the band.

Many of the experimental features of arbitrary-width confined states were also observed in the numerical simulations of the authors of Ref. [8]. For $\psi = -0.25$, they observed a wide confined state whose frequency and spatial structure matched experimental observations in detail [11]. Wide confined states were also observed in the coupled-field model of Ref. [9], for the case of a CGLE with purely real coefficients. Wide confined states drift forwards in Ref. [8] and backwards in Ref. [9], in contradiction to each other and to the experimental observations. A much more serious puzzle, however, is presented by the observation of a continuum of confined-state widths for each Rayleigh number in the locking band, because confined-state solutions of the single-field CGLE are known to form a countable, discrete set [3]. Locking cannot arise in this model, even if "nonadiabatic" effects are included [12].

These discrepancies, along with an advancing understanding of the experimental role of inhomogeneities [6,13], have raised the suspicion that the locking band and the lack of drift seen in Refs. [10,11] were experimental artifacts, caused by pinning of the confined-state boundaries by imperfections in the experimental cell. To resolve these issues, I have conducted new experiments on arbitrary-width confined states in the cell with greatly improved spatial uniformity [6,13]. I find that these confined states drift *backwards*. More importantly, while there is a hint of "locking" in the raw data, this quickly disappears when the data are corrected by subtracting off a smooth function of confined-state width. To within quite high precision, there is no "locking band."

The apparatus and techniques used in these experiments have been extensively described in recent publications [6,13]. The cell is an annular channel of height

$d = 0.2737$ cm, radial width $1.677d$, and mean circumference $82.47d$, formed by a concentric plastic disk and ring which are clamped between an electrically heated, mirror-polished silicon bottom plate and a transparent, water-cooled sapphire top plate. The fractional spatial uniformity and temporal stability of the applied vertical temperature difference are 4×10^{-4} and 5×10^{-5}, respectively. The flow pattern is viewed from above by a shadowgraphic flow-visualization system and is recorded by an annular array of 720 photodiodes. In the present experiments, this pattern consists of radial wave fronts which propagate around the cell in a single azimuthal direction under a spatially localized amplitude profile. This profile and the corresponding wave-number profile are extracted from the raw signal by demodulation in space at the measured mean wave number, using the techniques described in Ref. [13]. The precision of these calculations is at the 1% level. The fluid is an 8 wt % solution of ethanol in water at a mean temperature of 27.8 °C, for which the separation ratio $\psi = -0.253$, Prandtl number $P = 8.93$, and the Lewis number $L = 0.0079$ [14]. In this paper, lengths are scaled by the cell height d, and velocities are scaled by d/τ_v, where $\tau_v = 55.6$ sec is the vertical thermal diffusion time. The sign of velocities is defined by the direction of TW propagation.

TW convection is triggered in this system at $r_{co} = 1.459\,74(18)$ and persists down to $r_s = 1.2421(8)$ [15]. Inside this hysteresis loop, confined states can be created using techniques described in Refs. [6,10]. I have studied confined states at Rayleigh numbers in the range $1.328 \lesssim r \lesssim 1.349$. Most of the confined states studied had spatial widths in the range $8 < w < 12$, with a few data points taken at much larger widths. All of the confined states studied were narrower than half the circumference of the cell. A typical example of a wide confined state is shown in Fig. 1. This state exhibited a full width at half maximum spatial width of 21.1, and its amplitude and wave-number profiles match the measurements presented in Ref. [11]. The TW's in this state propagate to the left with phase velocity $v_{ph} = 1.480$, while the amplitude profile drifts in the opposite direction at a much lower velocity: $v_{dr} = -0.0237$. I have allowed confined states like this one to drift around the cell continually for weeks, taking approximately 60 h ($4000\tau_v$) for a round trip.

Quantitative measurements of the drift and the spatial growth of arbitrary-width confined states are presented in Fig. 2. To produce each data point in these graphs, a confined state was allowed to evolve for several hours at constant Rayleigh number, and then a series of flow-visualization images was recorded over the next several hours. The leading- and trailing-edge fronts of the confined state were identified at each time step as the 50% points in the demodulated TW amplitude profile. Differentiating in time to produce the front velocities v_l and v_t, respectively, I define the drift velocity $v_{dr} = (v_l + v_t)/2$ and the expansion velocity $\Delta v = v_l - v_t$. As shown in Fig. 2(a), the drift velocity depends very little on the Rayleigh number or on the confined-state width. The weighted average of the data points in Fig. 2(a) is $\bar{v}_{dr} = -0.0205(34)$. The expansion velocity, plotted in Fig. 2(b), increases with Rayleigh number, with a slope of about 3. The data exhibit substantial scatter; a fit to a cubic polynomial in r exhibits an rms residual of 0.0033. It might be thought at first that the slight flattening of the trend of the data seen in the range $1.335 \lesssim r \lesssim 1.340$ represents a locking band obscured by the scatter. However, both of these features are caused in fact not by locking but by the dependence of the expan-

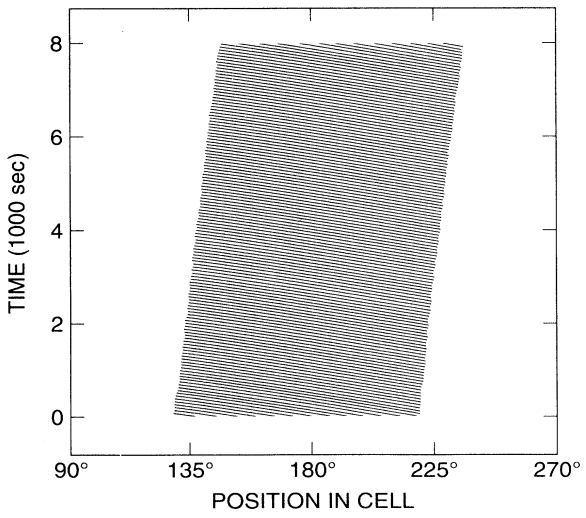

FIG. 1. Space-time paths of the convective roll boundaries are shown for a confined state of spatial width 21.1. The reduced Rayleigh number for this run was $r = 1.3402$. Rolls propagate to the left at a velocity $v_{ph} = 1.480$, while the confined state as a whole drifts to the right at a velocity $v_{dr} = -0.0237$.

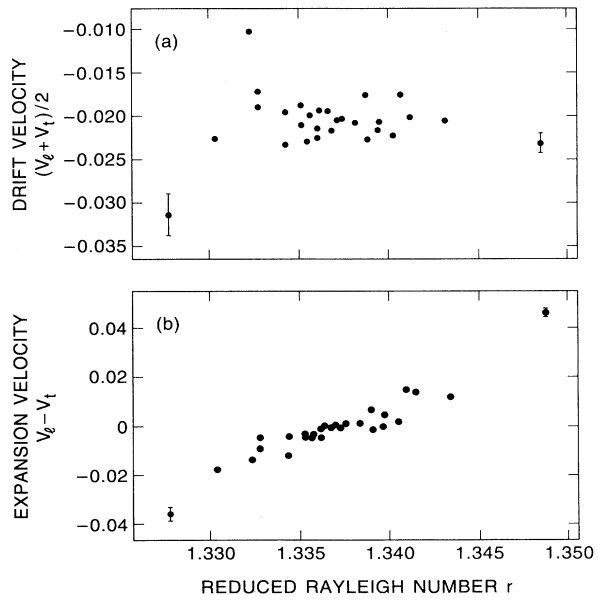

FIG. 2. Mean drift velocity $v_{dr} = (v_l + v_t)/2$ (a) and the expansion velocity $\Delta v = v_l - v_t$ (b) are plotted as functions of the reduced Rayleigh number r. In this and subsequent figures, error bars are shown when larger than the symbol.

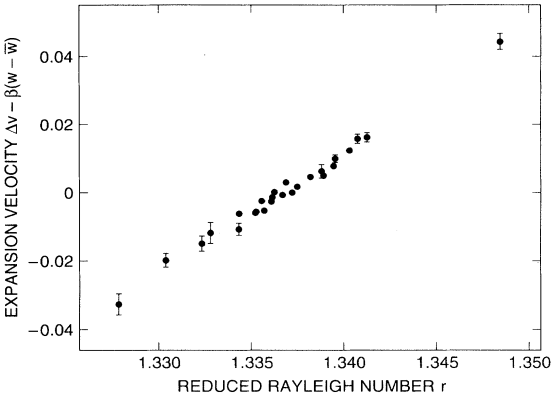

FIG. 3. Width-corrected expansion velocity $\Delta\bar{v}=\Delta v-\beta(w-\bar{w})$ is plotted as a function of the reduced Rayleigh number r. To within the experimental precision, $\Delta\bar{v}$ is a smooth function of r with a unique intercept $r_v=1.336\,48(23)$.

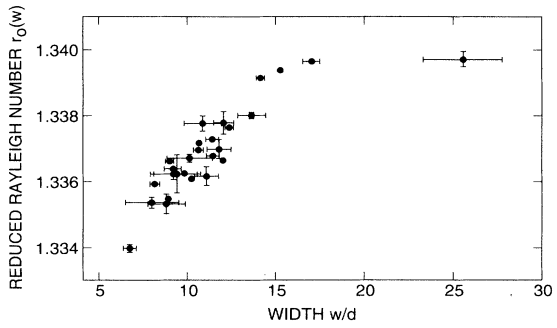

FIG. 4. Plotted as a function of the confined-state width w is the Rayleigh number $r_o(w)=r-\tau_o\Delta v(r,w)$ at which the expansion velocity vanishes. The data for $w\leq 17$ are well represented by a straight line of slope $dr_o/dw=4.8(3)\times10^{-4}$. $r_o(w)$ appears to increase very weakly with w for $w\gtrsim 20$.

sion of the confined state on its width w. I extracted this dependence by fitting the data points $\Delta v(r,w)$ to the sum of a cubic in r and a linear function $\beta(w-\bar{w})$, where $\bar{w}=10$ is the average experimental width. Using the fit parameter β, the width dependence of $\Delta v(r,w)$ can be removed by computing $\Delta\bar{v}(r)=\Delta v(r,w)-\beta(w-\bar{w})$ for each data point. This width-corrected expansion velocity is plotted vs Rayleigh number in Fig. 3. The error bars in this graph are somewhat larger than those in Fig. 2(b) because the confined-state width w is not constant in time during most of the measurements; this introduces uncertainty in the width correction $\beta(w-\bar{w})$. Nonetheless, the width correction has reduced the scatter in the data to 0.0015 rms and has revealed a smooth dependence of $\Delta\bar{v}$ on r. The fitted function of r passes through the horizontal axis with slope $\tau_o^{-1}=3.27(20)$ at an intercept $r_v=1.336\,48(23)$, which I interpret as the unique value at which a confined state of width $\bar{w}$ neither grows nor shrinks. To within this very high precision, there is no locking band.

To the extent that the width-corrected expansion velocity is linear in the Rayleigh number, confined-state evolution can be represented by a simple differential equation:

$$\tau_o\frac{dw}{dt}=r-r_o(w)\ , \tag{1}$$

where $r_o(w)=r_v-\tau_o\beta(w-\bar{w})$ is the Rayleigh number at which a confined state of width w neither grows nor shrinks. Using the fit parameter τ_o^{-1}, each measurement $\Delta v(r,w)$ yields a value of $r_o(w)$ which depends only weakly on the functional form assumed for the width dependence: $r_o(w)=r-\tau_o\Delta v(r,w)$. Figure 4 shows the dependence of this Rayleigh number on width. There appears to be a break in the slope of $r_o(w)$ near $w=17$; since there are not enough data points at large w to determine the shape of this function there, I excluded the data point at $w=25.5$ from the fit in the previous paragraph and from Fig. 3. Without this point, $r_o(w)$ is indeed an approximately linear function of width, with slope $dr_o/dw=4.8(3)\times10^{-4}$ for $7\lesssim w\lesssim 17$. Qualitative observations of very wide confined states suggest that dr_o/dw is very weakly positive for $25\lesssim w\lesssim 40$ [16].

The sign and magnitude of the derivative dr_o/dw are important because they determine the stability of the confined state: steady-state solutions to Eq. (1) are stable if $dr_o/dw>0$. In this regime, a confined state of width w simply relaxes to the time-independent width w_r given by $r_o(w_r)=r$. In the present experiments, I did not wait for this to happen. If I had done so, then the data graphed in Fig. 2(b) would have exhibited a plateau at $\Delta v=0$, which could have been misinterpreted as a locking band. The slight flattening of the trend of the data in the center of Fig. 2(b) is probably the effect of a partial approach to this equilibrium. Solutions of Eq. (1) for which $dr_o/dw<0$ are unstable. An unstable confined state will grow or shrink until it either fills the system, disappears completely, or attains a width for which $dr_o/dw>0$. At $\psi=-0.127$, it is possible to force fixed-width TW pulses to grow into arbitrary-width confined states which are globally and strongly unstable; i.e., for which $r_o(w)$ is a monotonically decreasing function and the expansion rate $|\tau_o^{-1}dr_o/dw|$ is large. These unstable confined states can be maintained in a steady state only by actively controlling their width with a servo. Observations of this type will be reported in the future.

The results of these experiments are that confined states at $\psi=-0.253$ drift backwards, do not exhibit locking, and are stable. The previous observations of stationary confined states and a rather wide locking band can undoubtedly be ascribed to pinning by inhomogeneities in the experimental cell. As in the case of fixed-width TW pulses observed at smaller $|\psi|$, this pinning apparently has little effect on the structure of the confined state. The observation that time-independent, "arbitrary-width" confined states actually have a unique width at a given Rayleigh number removes a long-standing discrepancy between the experimental observation of a locking band and the prediction of a discrete family of confined-state solutions for the CGLE model.

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[16] The "saturated" shape of the width dependence shown in Fig. 4 has been confirmed in detail in experiments using a fluid with $\psi = -0.210$.